Machine Fusion

Merging merges, more or less

Amos Robinson
Ambiata and UNSW (Australia)
amosr@cse.unsw.edu.au

Ben Lippmeier
Digital Asset and UNSW (Australia)
benl@ouroborus.net

ABSTRACT

Compilers for stream programs often rely on a fusion transformation to convert the implied dataflow network into low-level iteration based code. Different fusion transformations handle different sorts of networks, with the distinguishing criteria being whether the network may contain splits and joins, and whether the set of fusible operators can be extended. We present the first fusion system that simultaneously satisfies all three of these criteria: networks can contain splits, joins, and new operators can be added to the system without needing to modify the overall fusion transformation.

ACM Reference format:
Amos Robinson and Ben Lippmeier. 2017. Machine Fusion. In *Proceedings of ACM Conference, Washington, DC, USA, July 2017 (Conference'17),* 17 pages.
https://doi.org/10.1145/nnnnnnn.nnnnnnn

1 INTRODUCTION

Suppose we have two input streams of numeric identifiers, and wish to perform some analysis on them. The identifiers from both streams are sorted, but may include duplicates. We wish to produce an output stream of unique identifiers from the first input stream, as well as produce the unique union of both streams. Here is how we might write the source code, where `S` is for `S`-tream.

```
uniquesUnion : S Nat -> S Nat -> (S Nat, S Nat)
uniquesUnion sIn1 sIn2
 = let  sUnique = group sIn1
        sMerged = merge sIn1 sIn2
        sUnion  = group sMerged
   in   (sUnique, sUnion)
```

The `group` operator detects groups of consecutive identical elements and emits a single representative, while `merge` combines two sorted streams so that the output remains sorted. This example has a few interesting properties. Firstly, the data-access pattern of `merge` is *value-dependent*, meaning that the order in which `merge` pulls values from `sIn1` and `sIn2` depends on the values themselves: at each step, `merge` must compare the values from both streams, and choose the stream with the smaller value to pull from.

Secondly, although `sIn1` occurs twice in the program, at runtime we only want to handle the elements of each stream once. To achieve this, the compiled program must coordinate between the two uses of `sIn1`, so that a new value is read only when both the `group` and `merge` operators are ready to receive it. Finally, as the stream length is unbounded, we cannot buffer an arbitrary number of elements read from either stream, or we risk running out of local storage space.

Conference'17, July 2017, Washington, DC, USA
2017. ACM ISBN 978-x-xxxx-xxxx-x/YY/MM. . . $15.00
https://doi.org/10.1145/nnnnnnn.nnnnnnn

To implement this program we might write each operator as its own concurrent process, sending stream elements over intra-process channels. Developing this could be easy or hard, depending on the available language features for concurrency. However, the *performance tuning* of such a system, such as using back-pressure to prevent buffers from being overrun, or how to chunk stream data to amortize communication overhead, is invariably a headache.

Instead, we would prefer to use *stream fusion*, which is a program transformation that takes the implied dataflow network and produces a simple sequential loop that does not require extra process-control abstractions or unbounded buffering. Sadly, existing stream fusion transformations cannot handle our example.

As observed by Kay [15], both pull-based and push-based fusion have fundamental limitations. Pull-based systems such as short-cut stream fusion [9] cannot handle cases where a particular stream or intermediate result is used by multiple consumers. We refer to this situation as a *split* — in the dataflow network of our example the flow from input stream `sIn1` is split into both the `group` and `merge` consumers. Push-based systems such as foldr/build fusion [11] cannot fuse our example either, because they do not support operators with multiple inputs. We refer to this as a *join* — in our example the `merge` operator expresses a join in the data-flow network. Some systems support both pull and push: data flow inspired array fusion using series expressions [20] allows both splits and joins but only for a limited, predefined set of operators. More recent work on polarized data flow fusion [22] *is* able to fuse our example, but requires the program to be rewritten to use explicitly polarized stream types.

In this paper we present Machine Fusion, a new approach. Each operator is expressed as a restricted, sequential imperative program which *pulls* from input streams, and *pushes* to output streams. We view each operator as a process in a concurrent process network. Our fusion transform then *sequentializes* the concurrent process network into a single process, by choosing a particular interleaving of the operator code that requires no unbounded intermediate buffers. When the fusion transform succeeds we know it has worked. There is no need to inspect intermediate representations of the compiled code to debug poor performance, which is a common problem in systems based on general purpose program transformations [21].

In summary, we make the following contributions:

- a process calculus for infinite streaming programs (§2);
- a fusion algorithm, the first to support splits and joins (§4);
- benchmarks showing significant performance gains (§5);
- proof of soundness for the fusion algorithm in Coq (§6).

Our fusion transform for infinite stream programs also serves as the basis for an *array* fusion system, using a natural extension to finite streams. We discuss this extension in §5.1.

2 PROCESSES AND MACHINES

A *process* in our system is a simple imperative program with a local heap. A process pulls source values from an arbitrary number of input streams and pushes result values to at least one output stream. The process language is an intermediate representation we use when fusing the overall dataflow network. When describing the fusion transform we describe the control flow of the process as a state machine, hence Machine Fusion.

A *combinator* is a template for a process which parameterizes it over the particular input and output streams, as well as values of configuration parameters such as the worker function used in a `map` process. Each process implements a logical *operator* — so we use "operator" when describing the values being computed, but "process" and "machine" when referring to the implementation.

2.1 Grouping

The definition of the `group` combinator which detects groups of successive identical elements in the input stream is given in Figure 1. The process emits the first value pulled from the stream and every value that is different from the last one that was pulled. For example, when executed on the input stream `[1,2,2,3]`, the process will produce the output `[1,2,3]`. We include the concrete representation and a diagram of the process when viewed as a state machine.

The `group` combinator has two parameters, `sIn1` and `sOut1`, which bind the input and output streams respectively. The *nu-binders* (`ν (f: Bool) (l: Nat)`...) indicate that each time the `group` combinator is instantiated, fresh names must be given to `f`, `l` and so on, that do not conflict with other instantiations. Overall, the `f` variable tracks whether we are dealing with the first value from the stream, `l` holds the last value pulled from the stream (or 0 if none have been read yet), and `v` holds the current value pulled from the stream.

The body of the combinator is a record that defines the process. The `ins` field defines the set of input streams and the `outs` field the set of output streams. The `heap` field gives the initial values of each of the local variables. The `instrs` field contains a set of labeled instructions that define the program, while the `label` field gives the label of the initial instruction. In this form, the output stream is defined via a parameter, rather than being the result of the combinator, as in the representation of `uniquesUnion` from §1.

The initial instruction `(pull sIn1 v A1 [])` pulls the next element from the stream `sIn1`, writes it into the heap variable `v` (value), then proceeds to the instruction at label `A1`. The empty list `[]` after the target label `A1` can be used to update heap variables, but as we do not need to update anything yet we leave it empty.

Next, the instruction `(case (f || (l /= v)) A2 [] A3 [])` checks whether predicate `(f || (l /= v))` is true; if so it proceeds to the instruction at label `A2`, otherwise it proceeds to `A3`. We use the variable `l` (last) to track the last value read from the stream, and the boolean `f` (first) to track whether this is the first element.

When the predicate is true, the instruction at label `A2` executes `(push sOut1 v A3 [ l = v, f = F ])` which pushes the value `v` to the output stream `sOut1` and proceeds to the instruction at label `A3`, once the variable `l` is set to `v` and `f` to `F` (False).

Finally, the instruction `(drop sIn1 A0 [])` signals that the current element that was pulled from stream `sIn1` is no longer required, and goes back to the first instruction at `A0`.

2.2 Merging

The definition of the `merge` combinator, which merges two input streams, is given in Figure 2. The combinator binds the two input streams to `sIn1` and `sIn2`, while the output stream is `sOut2`. The two heap variables `x1` and `x2` store the last values read from each input stream. The process starts by pulling from each of the input streams. It then compares the two pulled values, and pushes the smaller of the values to the output stream. The process then drops the stream which yielded the the smaller value, then pulls from the same stream so that it can perform the comparison again.

2.3 Fusion

Our fusion algorithm takes two processes and produces a new one that computes the output of both. For example, suppose we need a single process that produces the output of the first two lines of our `uniquesUnion` example back in §1. The result will be a process that computes the result of both `group` and `merge` as if they were executed concurrently, where the first input stream of the `merge` process is the same as the input stream of the `group` process. In our informal description of the fusion algorithm we will instantiate the parameters of each combinator with arguments of the same names.

2.3.1 Fusing Pulls. The algorithm proceeds by considering pairs of states: one from each of the source process state machines to be fused. Both the `group` machine and the `merge` machine pull from the same stream as their initial instruction, so we have the situation shown in the top of Figure 3. The `group` machine needs to transition from label `A0` to label `A1`, and the `merge` machine from `B0` to `B1`. In the result machine we produce three new instructions that transition between four joint result states, `F0` to `F3`. Each of the joint result states represents a combination of two source states, one from each of the source machines. For example, the first result state `F0` represents a combination of the `group` machine being in its initial state `A0` and the `merge` machine being in its own initial state `B0`.

We also associate each of the joint result states with a description of whether each source machine has already pulled a value from each of its input streams. For the `F0` case at the top of Figure 3 we have ((A0, {sIn1 = none}), (B0, {sIn1 = none, sIn2 = none})). The result state `F0` represents a combination of the two source states `A0` and `B0`. As both `A0` and `B0` are the initial states of their respective machines, those machines have not yet pulled any values from their two input streams, so both 'sIn1' and 'sIn2' map to 'none'.

From the result state `F0`, both of the source machines then need to pull from stream `sIn1`, the `group` machine storing the value in a variable `v` and the `merge` machine storing it in `x1`. In the result machine this is managed by first storing the pulled value in a fresh, shared buffer variable `b1`, and then using later instructions to copy the value into the original variables `v` and `x1`. To perform the copies we attach updates to a `jump` instruction, which otherwise transitions between states without affecting any of the input or output streams.

Finally, note that in the result states `F0` through `F3`, the state of the input streams transitions from 'none', to 'pending' then to 'have'. The 'none' state means that we have not yet pulled a value from the associated stream. The 'pending' state means we have pulled a value into the stream buffer variable (`b1` in this case). The 'have' state means that we have copied the pulled value from the stream buffer variable into the local variable used by each machine.

```
group
  = λ (sIn1: Stream Nat) (sOut1: Stream Nat).
    ν (f: Bool) (l: Nat) (v: Nat) (A0..A3: Label).
    process
     { ins:    { sIn1  }
     , outs:   { sOut1 }
     , heap:   { f = T, l = 0, v = 0 }
     , label:  A0
     , instrs: { A0 = pull sIn1 v          A1 []
               , A1 = case (f || (l /= v)) A2 []  A3 []
               , A2 = push sOut1 v         A3 [ l = v, f = F ]
               , A3 = drop sIn1            A0 [] } }
```

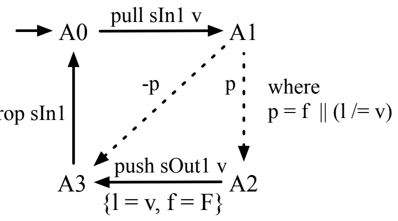


Figure 1: The group combinator

```
merge
  = λ (sIn1: Stream Nat) (sIn2: Stream Nat) (sOut2: Stream Nat).
    ν (x1: Nat) (x2: Nat) (B0..E2: Label).
    process
     { ins:    { sM1, sM2 }
     , outs:   { sM3 }
     , heap:   { x1 = 0, x2 = 0 }
     , label:  B0
     , instrs: { B0 = pull sIn1  x1   B1 []                , B1 = pull sIn2  x2   C0 []
               , C0 = case (x1 < x2)  D0 []  E0 []         , D0 = push sOut2 x1   D1 []
               , D1 = drop sIn1       D2 []                , D2 = pull sIn1  x1   C0 []
               , E0 = push sOut2 x2   E1 []                , E1 = drop sIn2       E2 []
               , E2 = pull sIn2 x2    C0 [] } }
```

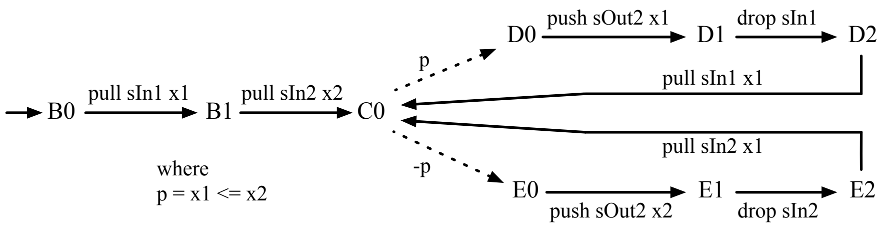


Figure 2: The merge combinator

2.3.2 Fusing Cases. Once the result machine has arrived at the joint state F3, this is equivalent to the two source machines arriving in states A1 and B1 respectively. The lower half of Figure 3 shows the next few transitions of the source machines. From state A1, the group machine needs to perform a case branch to determine whether to push the current value it has from its input stream sIn1 to output stream sOut1, or to just pull the next value from its input. From state B1, the merge machine needs to pull a value from its second input stream sIn2. In the result machine, F3 performs the case analysis from A1, moving to either F4 or F5, corresponding to A2 and A3 respectively. From state F4, the push at A2 is executed and moves to F5, corresponding to A3. Finally, at F5 the merge machine pulls from sIn2, moving from F5 to F6, corresponding to B1 and C0 respectively. As the stream sIn2 is only pulled from by the merge machine, no coordination is required between the merge and group machines for this pull.

2.4 Fused Result

Figure 4 shows the final result of fusing group and merge together. There are similar rules for handling the other combinations of instructions, but we defer the details to §4. The result process has two input streams, sIn1 and sIn2, and two output streams: sOut1 from group, and sOut2 from merge. The shared input sIn1 is pulled by merge instructions at two places, and since both of these need to agree with when group pulls, the group instructions are duplicated at F3-F5 and F13-F15. The first set of instructions could be simplified by constant propagation to a single push, as f is initially true.

To complete the implementation of our example from §1 we would now fuse this result process with a process from the final line of the example (also a group). Note that although the result process has a single shared heap, the heap bindings from each fused process are guaranteed not to interfere, as when we instantiate combinators to create source processes we introduce fresh names.

2.5 Breaking It Down

We started with a pure functional program in §1, reimagined it as a dataflow graph, then interleaved imperative code that implemented two of the operators in that dataflow graph. We needed to *break down* the definition of each operator into imperative statements so that we could interleave their execution appropriately. We do this because the standard, single-threaded evaluation semantics of functional programs does not allow us to evaluate stream programs that contain both splits and joins in a space efficient way. Returning to the definition of `uniquesUnion` from §1, we cannot simply execute the `group` operator on its entire input `sIn1` before executing the `merge` operator, as that would require us to buffer all data read from `sIn1`. Instead, during fusion we perform the job of a concurrent scheduler at compile time. In the result process the flow of control alternates between the instructions for both the `group` and `merge` operators, but as the instructions are interleaved directly there is no overhead due to context switching — as there would be in a standard concurrent implementation using multiple threads.

The general approach of converting a pure functional program to a dataflow graph, then interleaving imperative statements that implement each operator was also used in prior work on Flow Fusion [20]. However, in contrast to Flow Fusion and similar systems, with Machine Fusion we do not need to organize statements into a fixed *loop anatomy* — we simply merge them as they are. This allows us to implement a wider range of operators, including ones with nested loops that work on segmented streams.

Note that relying on lazy evaluation for `uniquesUnion` does not eliminate the need for unbounded buffering. Suppose we converted each of the streams to lazy lists, and used definitions of `group` and `merge` that worked over these lists. As `uniquesUnion` returns a pair of results, there is nothing preventing a consumer from demanding the first list (`sUnique`) in its entirety before demanding any of the elements from the second list (`sUnion`). If this were to happen then the runtime implementation would need to retain all elements of `sIn1` before demanding any of `sIn2`, causing a space leak. Lazy evaluation is *pull only* meaning that evaluation is driven by the consumer. The space efficiency of our fused program relies critically on the fact that processes can also *push* their result values directly to their consumers, and that the consumers cannot defer the handling of these values.

3 PROCESS DEFINITIONS

The formal grammar for process definitions is given in Figure 5. Variables, Channels and Labels are specified by unique names. We refer to the *endpoint* of a stream as a channel. A particular stream may flow into the input channels of several different processes, but can only be produced by a single output channel. For values and expressions we use an untyped lambda calculus with a few primitives chosen to facilitate the examples. The '||' operator is boolean-or, '+' addition, '/=' not-equal, and '<' less-than.

A *Process* is a record with five fields: the `ins` field specifies the input channels; the `outs` field the output channels; the `heap` field the process-local heap; the `label` field the label of the instruction currently being executed, and the `instrs` a map of labels to instructions. We use the same record when specifying both the definition of a particular process, as well as when giving the evaluation semantics. When specifying a process the `label` field gives the entry-point to the process code, though during evaluation it is the label of the instruction currently being executed. Likewise, when specifying a process we usually only list channel names in the `ins` field, though during evaluation they are also paired with their current *InputState*. If an *InputState* is not specified we assume it is 'none'. A network is a set of processes that are able to communicate with each other.

In the grammar of Figure 5 the *InputState* has three options: `none`, which means no value is currently stored in the associated stream buffer variable, (`pending` *Value*) which gives the current value in the stream buffer variable and indicates that it has not yet been copied into a process-local variable, and `have` which means the pending value has been copied into a process-local variable. The *Value* attached to the `pending` state is used when specifying the evaluation semantics of processes. When performing the fusion transform the *Value* itself will not be known, but we can still reason statically that a process must be in the `pending` state. When defining the fusion transform in §4 we will use a version of *InputState* with only this statically known information.

The `instrs` field of the *Process* maps labels to instructions. The possible instructions are: `pull`, which pulls the next value from a channel into a given heap variable; `push`, which pushes the value of an expression to an output channel; `drop` which indicates that the current value pulled from a channel is no longer needed; `case` which branches based on the result of a boolean expression, and `jump` which causes control to move to a new instruction.

Instructions include a *Next* field containing the label of the next instruction to execute, as well as a list of $Variable \times Exp$ bindings used to update the heap. The list of update bindings is attached directly to instructions to make the fusion algorithm easier to specify, in contrast to a presentation with a separate `update` instruction.

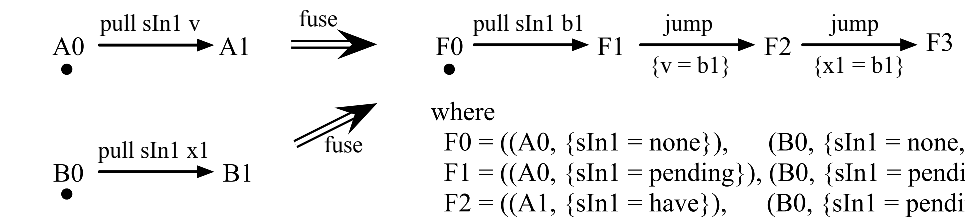
When lowering process code to a target language, such as C, LLVM, or some sort of assembly code, we can safely convert `drop` to plain `jump` instructions. The `drop` instructions are used to control how processes should be synchronized, but do not affect the execution of a single process. We discuss `drop`s further in §5.3.

3.1 Execution

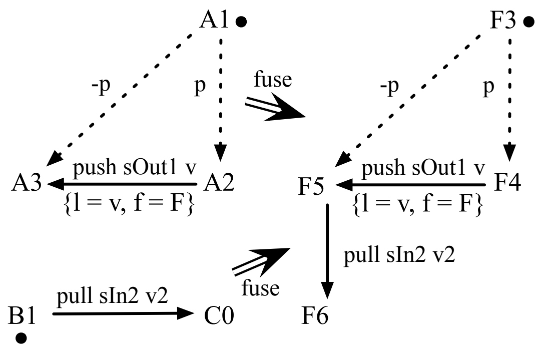
The dynamic execution of a process network consists of:

(1) *Injection* of a single value from a stream into a process, or a network. Each individual process only needs to accept an injected value when it is ready for it, and injection into a network succeeds only when they *all* processes accept it.
(2) *Advancing* a single process from one state to another. Advancing a network succeeds when *any* of the processes in the network can advance.
(3) *Feeding* outputs of some processes to the inputs of others. Feeding alternates between Injecting and Advancing. When a process pushes a value to an output channel we attempt to inject this value into all processes that have that same channel as an input. If they all accept it, we then advance their programs as far as they will go, which may cause more values to be pushed to output channels, and so on.

Execution of a network is non-deterministic. At any moment several processes may be able to take a step, while others are blocked. As with Kahn processes [14], pulling from a channel is blocking,



where
F0 = ((A0, {sIn1 = none}), (B0, {sIn1 = none, sIn2 = none}))
F1 = ((A0, {sIn1 = pending}), (B0, {sIn1 = pending, sIn2 = none}))
F2 = ((A1, {sIn1 = have}), (B0, {sIn1 = pending, sIn2 = none}))
F3 = ((A1, {sIn1 = have}), (B1, {sIn1 = have, sIn2 = none}))



where
p = f || (l /= v)
F3 = ((A1,{sIn1 = have}), (B1,{sIn1 = have, sOut2 = none}))
F4 = ((A2,{sIn1 = have}), (B1,{sIn1 = have, sOut2 = none}))
F5 = ((A3,{sIn1 = have}), (B1,{sIn1 = have, sOut2 = none}))
F6 = ((A3,{sIn1 = have}), (C0,{sIn1 = have, sOut2 = have}))

Figure 3: Fusing pull (top) and case (bottom) instructions

```
process
{ ins:    { sIn1,  sIn2 }
, outs:   { sOut1, sOut2 }
, heap:   { f = T, l = 0, v = 0, x1 = 0, x2 = 0, b1 = 0 }
, label:  F0
, instrs:
  { F0  = pull sIn1 b1          F1  [  ]              F0   = ((A0,{sIn1 = none}),    (B0, {sIn1 = none,    sIn2 = none}))
  , F1  = jump                  F2  [ v = b1 ]        F1   = ((A0,{sIn1 = pending}), (B0, {sIn1 = pending, sIn2 = none}))
  , F2  = jump                  F3  [ x1 = b1 ]       F2   = ((A1,{sIn1 = have}),    (B0, {sIn1 = pending, sIn2 = none}))
  , F3  = case (f || (l /= v))  F4  [  ]    F5  [  ]  F3   = ((A1,{sIn1 = have}),    (B1, {sIn1 = have,    sIn2 = none}))
  , F4  = push sOut1 v          F5  [ l = v, f = F ]  F4   = ((A2,{sIn1 = have}),    (B1, {sIn1 = have,    sIn2 = none}))
  , F5  = jump                  F6  [  ]              F5   = ((A3,{sIn1 = have}),    (B1, {sIn1 = have,    sIn2 = none}))
  , F6  = pull sIn2 x2          F7  [  ]              F6   = ((A0,{sIn1 = none}),    (B1, {sIn1 = have,    sIn2 = none}))

  , F7  = case (x1 < x2)        F8  [  ]    F16 [  ]  F7   = ((A0,{sIn1 = none}),    (C0, {sIn1 = have,    sIn2 = have}))

  , F8  = push sOut2 x1         F9  [  ]              F8   = ((A0,{sIn1 = none}),    (D0, {sIn1 = have,    sIn2 = have}))
  , F9  = drop sIn1             F10 [  ]              F9   = ((A0,{sIn1 = none}),    (D1, {sIn1 = none,    sIn2 = have}))
  , F10 = pull sIn1 b1          F11 [  ]              F10  = ((A0,{sIn1 = none}),    (D2, {sIn1 = none,    sIn2 = have}))
  , F11 = jump                  F12 [ v = b1 ]        F11  = ((A0,{sIn1 = pending}), (D2, {sIn1 = pending, sIn2 = have}))
  , F12 = jump                  F13 [ x1 = b1 ]       F12  = ((A1,{sIn1 = have}),    (D2, {sIn1 = pending, sIn2 = have}))
  , F13 = case (f || (l /= v))  F14 [  ]    F15 [  ]  F13  = ((A1,{sIn1 = have}),    (C0, {sIn1 = have,    sIn2 = have}))
  , F14 = push sOut1 v          F15 [ l = v, f = F ]  F14  = ((A2,{sIn1 = have}),    (C0, {sIn1 = have,    sIn2 = have}))
  , F15 = jump                  F7  [  ]              F15  = ((A3,{sIn1 = have}),    (C0, {sIn1 = have,    sIn2 = have}))

  , F16 = push sOut2 x2         F17 [  ]              F16  = ((A0,{sIn1 = none}),    (E0, {sIn1 = have,    sIn2 = have}))
  , F17 = drop sIn2             F18 [  ]              F17  = ((A0,{sIn1 = none}),    (E1, {sIn1 = have,    sIn2 = have}))
  , F18 = pull sIn2             F7  [  ]              F18  = ((A0,{sIn1 = none}),    (E2, {sIn1 = have,    sIn2 = none}))
} }
```

Figure 4: Fusion of group and merge, along with shared instructions

$$\begin{array}{lll}
\mathit{Exp}, e & ::= & x \mid v \mid e\ e \\
 & \mid & (e \mid\mid e) \mid e + e \mid e \mathrel{/=} e \mid e < e \\
\mathit{Value}, v & ::= & \mathbb{N} \mid \mathbb{B} \mid (\lambda x.\ e) \\
\mathit{Heap}, \mathit{bs} & ::= & \cdot \mid \mathit{bs},\ x = v \\
\mathit{Updates}, \mathit{us} & ::= & \cdot \mid \mathit{us},\ x = e \\
\\
\mathit{Process}, p & ::= & \mathtt{process} \\
 & & \quad \mathtt{ins:}\ \ (\mathit{Channel} \mapsto \mathit{InputState}) \\
 & & \quad \mathtt{outs:}\ \ \{\mathit{Channel}\} \\
 & & \quad \mathtt{heap:}\ \ \mathit{Heap} \\
 & & \quad \mathtt{label:}\ \ \mathit{Label} \\
 & & \quad \mathtt{instrs:}\ (\mathit{Label} \mapsto \mathit{Instruction}) \\
\\
\mathit{InputState} & ::= & \mathtt{none} \mid \mathtt{pending}\ \mathit{Value} \mid \mathtt{have}
\end{array}$$

$$\begin{array}{lllll}
\mathit{Variable}, x & \to & \multicolumn{3}{l}{\text{(value variable)}} \\
\mathit{Channel}, c & \to & \multicolumn{3}{l}{\text{(channel name)}} \\
\mathit{Label}, l & \to & \multicolumn{3}{l}{\text{(label name)}} \\
\mathit{ChannelStates} & = & \multicolumn{3}{l}{(\mathit{Channel} \mapsto \mathit{InputState})} \\
\mathit{Action}, a & ::= & \multicolumn{3}{l}{\cdot \mid \mathtt{push}\ \mathit{Channel}\ \mathit{Value}} \\
\\
\mathit{Instruction} & ::= & \mathtt{pull}\ \mathit{Channel} & \mathit{Variable} & \mathit{Next} \\
 & \mid & \mathtt{push}\ \mathit{Channel} & \mathit{Exp} & \mathit{Next} \\
 & \mid & \mathtt{drop}\ \mathit{Channel} & & \mathit{Next} \\
 & \mid & \mathtt{case}\ \mathit{Exp} & \mathit{Next} & \mathit{Next} \\
 & \mid & \mathtt{jump} & & \mathit{Next} \\
\\
\mathit{Next} & = & \multicolumn{3}{l}{\mathit{Label} \times \mathit{Updates}}
\end{array}$$

Figure 5: Process definitions

which enables the overall sequence of values on each output channel to be deterministic. Unlike Kahn processes, pushing to a channel can also block. Each consumer has a single element buffer, and pushing only succeeds when that buffer is empty.

Importantly, it is the order in which values are *pushed to each particular output channel* which is deterministic, whereas the order in which different processes execute their instructions is not. When we fuse two processes we choose one particular instruction ordering that enables the network to advance without requiring unbounded buffering. The single ordering is chosen by heuristically deciding which pair of states to merge during fusion, and is discussed in §3.2.

Each channel may be pushed to by a single process only, so in a sense each output channel is owned by a single process. The only intra-process communication is via channels and streams. Our model is "pure data flow" as there are no side-channels between processes — in contrast to "impure data flow" systems such as StreamIt [29].

3.1.1 *Injection.*

Figure 6 gives the rules for injecting values into processes. Injection is a meta-level operation, in contrast to `pull` and `push` which are instructions in the object language. The statement (p; `inject` $v\ c\ \Rightarrow\ p'$) reads "given process p, injecting value v into channel c yields an updated process p'". The `injects` form is similar, operating on a process network.

Rule (InjectValue) injects a single value into a single process. The value is stored as a (`pending` v) binding in the *InputState* of the associated channel of the process. The *InputState* acts as a single element buffer, and must be empty (`none`) for injection to succeed.

Rule (InjectIgnore) allows processes that do not use a particular named channel to ignore values injected into that channel.

Rule (InjectMany) attempts to inject a single value into a network. We use the single process judgment form to inject the value into all processes, which must succeed for all of them.

3.1.2 *Advancing.*

Figure 7 gives the rules for advancing a single process. The statement (i; is; $bs \overset{a}{\Rightarrow} l$; is'; us') reads "instruction i, given channel states is and the heap bindings bs, passes control to instruction at label l and yields new channel states is', heap update expressions us', and performs an output action a." An output action a is an optional message of the form (`push` *Channel Value*), which encodes the value a process pushes to one of its output channels. We write $\cdot$ for an empty action.

Rule (Pull) takes the `pending` value v from the channel state and produces a heap update to copy this value into the variable x in the `pull` instruction. We use the syntax $us, x = v$ to mean that the list of updates us is extended with the new binding $x = v$. In the result channel states, the state of the channel c that was pulled from is set to `have`, to indicate the value has been copied into the local variable.

Rule (Push) evaluates the expression e under heap bindings bs to a value v, and produces a corresponding action which carries this value. The judgment ($bs \vdash e \Downarrow v$) expresses standard untyped lambda calculus reduction using the heap bs for the values of free variables. As this evaluation is completely standard we omit it to save space.

Rule (Drop) changes the input channel state from `have` to `none`. A `drop` instruction can only be executed after `pull` has set the input channel state to `have`.

Rule (Jump) produces a new label and associated update expressions. Rules (CaseT) and (CaseF) evaluate the scrutinee e and emit the appropriate label.

The statement $p \overset{a}{\Rightarrow} p'$ reads "process p advances to new process p', yielding action a". Rule (Advance) advances a single process. We look up the current instruction for the process' `label` and pass it, along with the channel states and heap, to the above single instruction judgment. The update expressions us from the single instruction judgment are reduced to values before updating the heap. We use ($us \triangleleft bs$) to replace bindings in us with new ones from bs. As the update expressions are pure, the evaluation can be done in any order.

3.1.3 *Feeding.*

Figure 8 gives the rules for collecting output actions and feeding the contained values to other processes. The first set of rules concerns feeding values to other processes within the same network, while the second exchanges input and output values with the environment the network is running in.

The statement $ps \overset{a}{\Rightarrow} ps'$ reads "the network ps advances to the network ps' yielding output action a".

Rule (ProcessInternal) allows an arbitrary process in the network to advance to a new state at any time, provided it does not yield an output action. This allows processes to perform internal computation.

Rule (ProcessPush) allows an arbitrary process in the network to advance to a new state while yielding an output action (`push` $c\ v$). For this to succeed it must be possible to inject the output value v into all processes that have channel c as one of their inputs.

$$\boxed{\mathit{Process};\ \texttt{inject}\ \mathit{Value}\ \mathit{Channel}\ \Rightarrow\ \mathit{Process}} \quad \boxed{\{\mathit{Process}\};\ \texttt{injects}\ \mathit{Value}\ \mathit{Channel}\ \Rightarrow\ \{\mathit{Process}\}}$$

$$\frac{p[\texttt{ins}][c] = \texttt{none}}{p;\ \texttt{inject}\ v\ c\ \Rightarrow\ p\ [\texttt{ins} \mapsto (p[\texttt{ins}][c \mapsto \texttt{pending}\ v])]}\ (\text{InjectValue})$$

$$\frac{c \notin p[\texttt{ins}]}{p;\ \texttt{inject}\ v\ c\ \Rightarrow\ p}\ (\text{InjectIgnore}) \qquad \frac{\{\ p_i;\ \texttt{inject}\ v\ c\ \Rightarrow\ p'_i\ \}^i}{\{p_i\}^i;\ \texttt{injects}\ v\ c\ \Rightarrow\ \{p'_i\}^i}\ (\text{InjectMany})$$

Figure 6: Injection of values into input channels

$$\boxed{\mathit{Instruction};\ \mathit{ChannelStates};\ \mathit{Heap}\ \xRightarrow{\mathit{Action}}\ \mathit{Label};\ \mathit{ChannelStates};\ \mathit{Updates}}$$

$$\frac{is[c] = \texttt{pending}\ v}{\texttt{pull}\ c\ x\ (l, us);\ is;\ bs\ \dot{\Rightarrow}\ l;\ is[c \mapsto \texttt{have}];\ (us, x = v)}\ (\text{Pull}) \qquad \frac{bs \vdash e \Downarrow v}{\texttt{push}\ c\ e\ (l, us);\ is;\ bs\ \xRightarrow{\texttt{push}\ c\ v}\ l;\ is;\ us}\ (\text{Push})$$

$$\frac{is[c] = \texttt{have}}{\texttt{drop}\ c\ (l, us);\ is;\ bs\ \dot{\Rightarrow}\ l;\ is[c \mapsto \texttt{none}];\ us}\ (\text{Drop}) \qquad \frac{}{\texttt{jump}\ (l, us);\ is;\ bs\ \dot{\Rightarrow}\ l;\ is;\ us}\ (\text{Jump})$$

$$\frac{bs \vdash e \Downarrow \texttt{True}}{\texttt{case}\ e\ (l_t, us_t)\ (l_f, us_f);\ is;\ bs\ \dot{\Rightarrow}\ l_t;\ is;\ us_t}\ (\text{CaseT}) \qquad \frac{bs \vdash e \Downarrow \texttt{False}}{\texttt{case}\ e\ (l_t, us_t)\ (l_f, us_f);\ is;\ bs\ \dot{\Rightarrow}\ l_f;\ is;\ us_f}\ (\text{CaseF})$$

$$\boxed{\mathit{Process}\ \xRightarrow{\mathit{Action}}\ \mathit{Process}}$$

$$\frac{p[\texttt{instrs}][p[\texttt{label}]];\ p[\texttt{ins}];\ p[\texttt{heap}]\ \overset{a}{\Rightarrow}\ l;\ is;\ us \qquad p[\texttt{heap}] \vdash us \Downarrow bs}{p\ \overset{a}{\Rightarrow}\ p\ [\texttt{label}\ \mapsto\ l,\ \texttt{heap}\ \mapsto (p[\texttt{heap}] \triangleleft bs),\ \texttt{ins}\ \mapsto\ is]}\ (\text{Advance})$$

Figure 7: Advancing processes

$$\boxed{\{\mathit{Process}\}\ \xRightarrow{\mathit{Action}}\ \{\mathit{Process}\}}$$

$$\frac{p_i\ \dot{\Rightarrow}\ p'_i}{\{p_0 \ldots p_i \ldots p_n\}\ \dot{\Rightarrow}\ \{p_0 \ldots p'_i \ldots p_n\}}\ (\text{ProcessesInternal}) \qquad \frac{p_i\ \xRightarrow{\texttt{push}\ c\ v}\ p'_i \quad \forall j \mid j \neq i.\ p_j;\ \texttt{inject}\ c\ v\ \Rightarrow\ p'_j}{\{p_0 \ldots p_i \ldots p_n\}\ \xRightarrow{\texttt{push}\ c\ v}\ \{p'_0 \ldots p'_i \ldots p'_n\}}\ (\text{ProcessesPush})$$

$$\boxed{(\mathit{Channel} \mapsto \overline{\mathit{Value}})\ ;\ \{\mathit{Process}\}\ \Rightarrow\ (\mathit{Channel} \mapsto \overline{\mathit{Value}})\ ;\ \{\mathit{Process}\}}$$

$$\frac{ps\ \dot{\Rightarrow}\ ps'}{cvs;\ ps\ \Rightarrow\ cvs;\ ps'}\ (\text{FeedInternal}) \qquad \frac{ps\ \xRightarrow{\texttt{push}\ c\ v}\ ps'}{cvs;\ ps\ \Rightarrow\ cvs[c \mapsto (cvs[c]\ {+}{+}\ v)];\ ps'}\ (\text{FeedPush})$$

$$\frac{(\forall p \in ps.\ c \notin p[\texttt{outs}]) \qquad ps;\ \texttt{injects}\ c\ v\ \Rightarrow\ ps'}{cvs[c \mapsto ([v]\ {+}{+}\ vs)];\ ps\ \Rightarrow\ cvs[c \mapsto vs\ ];\ ps'}\ (\text{FeedExternal})$$

Figure 8: Feeding Process Networks

The statement $cvs;\ ps\ \Rightarrow\ cvs';\ ps'$ reads "with channel values cvs, network ps takes a step and produces new channel values cvs' and network ps'". The channel values cvs map channel names to a list of values. For input channels of the overall network, we initialize the map to contain a list of input values for each channel. For output channels of the overall network, values pushed to those channels are also collected in the same channel map. In a concrete implementation the input and output values would be transported over some IO device, but for the semantics we describe the abstract behavior only.

Rule (FeedInternal) allows the network to perform local computation in the context of the channel values.

Rule (FeedPush) collects an output action ($\texttt{push}\ c\ v$) produced by a network and appends the value v to the list corresponding to the output channel c.

Rule (FeedExternal) injects values from the external environment. This rule also has the side condition that values cannot be injected from the environment into output channels that are already owned by some process. This constraint is required for correctness proofs, but can be ensured by construction in a concrete implementation.

3.2 Non-deterministic Execution Order

The execution rules of Figure 8 are non-deterministic in several ways. Rule (ProcessInternal) allows any process to perform internal computation at any time, without synchronizing with other processes in the network; (ProcessPush) allows any process to perform a push action at any time, provided all other processes in the network are ready to accept the pushed value; (FeedExternal) also allows new values to be injected from the environment, provided all processes that use the channel are ready to accept the value.

In the semantics, allowing the execution order of processes to be non-deterministic is critical, as it defines a search space where we might find an order that does not require unbounded buffering. For a direct implementation of concurrent processes using message passing and operating system threads, an actual, working, execution order would be discovered dynamically at runtime. In contrast, the role of our fusion system is to construct one of these working orders statically. In the fused result process, the instructions will be scheduled so that they run in one of the orders that would have arisen if the network were executed dynamically. Fusion also eliminates the need to pass messages between processes — once they are fused we can just copy values between heap locations.

4 FUSION

Our core fusion algorithm constructs a static execution schedule for a single pair of processes. To fuse a whole process network we fuse successive pairs of processes until only one remains.

Figure 9 defines some auxiliary grammar used during fusion. We extend the *Label* grammar with a new alternative, *LabelF* × *LabelF* for the labels in a fused result process. Each *LabelF* consists of a *Label* from a source process, paired with a map from *Channel* to the statically known part of that channel's current *InputState*. When fusing a whole network, as we fuse pairs of individual processes the labels in the result collect more and more information. Each label of the final, completely fused process encodes the joint state that all the original source processes would be in at that point.

We also extend the existing *Variable* grammar with a (`chan` c) form which represents the buffer variable associated with channel c. We only need one buffer variable for each channel, and naming them like this saves us from inventing fresh names in the definition of the fusion rules. We used a fresh name back in §2.3.1 to avoid introducing a new mechanism at that point in the discussion.

Still in Figure 9, *ChannelType2* classifies how channels are used, and possibly shared, between two processes. Type `in2` indicates that the two processes `pull` from the same channel, so these actions must be coordinated. Type `in1` indicates that only a single process pulls from the channel. Type `in1out1` indicates that one process pushes to the channel and the other pulls. Type `out1` indicates that the channel is pushed to by a single process. Each output channel is uniquely owned and cannot be pushed to by more than one process.

Figure 10 defines function *fusePair* that fuses a pair of processes, constructing a result process that does the job of both. We start with a joint label l_0 formed from the initial labels of the two source processes. We then use *tryStepPair* to statically choose which of the two processes to advance, and hence which instruction to execute next. The possible destination labels of that instruction (computed with *outlabels* from Figure 13) define new joint labels and reachable

```
Label         ::= ... | LabelF × LabelF | ...
LabelF         =  Label × (Channel ↦ InputStateF)
InputStateF   ::= none_F | pending_F | have_F
Variable      ::= ... | chan Channel | ...
ChannelType2  ::= in2 | in1 | in1out1 | out1
```

Figure 9: Fusion type definitions.

```
fusePair : Process → Process → Maybe Process
fusePair p q
  | Just is ← go {} l_0
  = Just (process
          ins:   {c | c = t ∈ cs, t ∈ {in1, in2}}
          outs:  {c | c = t ∈ cs, t ∈ {in1out1, out1}}
          heap:  p[heap] ∪ q[heap]
          label: l_0
          instrs: is)
  | otherwise = Nothing
 where
  cs  = channels p q
  l_0 = ( (p[label], {c = none_F | c ∈ p[ins]})
        , (q[label], {c = none_F | c ∈ q[ins]}))
  go bs (l_p, l_q)
      | (l_p, l_q) ∈ bs
      = Just bs
      | Just b ← tryStepPair cs l_p p[instrs][l_p] l_q q[instrs][l_q]
      = foldM go (bs ∪ {(l_p, l_q) = b}) (outlabels b)
      | otherwise = Nothing
```

Figure 10: Fusion of pairs of processes

```
tryStepPair : (Channel ↦ ChannelType2)
            → LabelF → Instruction → LabelF → Instruction
            → Maybe Instruction
tryStepPair cs l_p i_p l_q i_q =
  match (tryStep cs l_p i_p l_q, tryStep cs l_q i_q l_p) with
  (Just i'_p, Just i'_q)
   | jump _ ← i'_p       → Just i'_p                 (PreferJump1)
   | jump _ ← i'_q       → Just (swaplabels i'_q)    (PreferJump2)
   | pull _ _ _ ← i'_q   → Just i'_p                 (DeferPull1)
   | pull _ _ _ ← i'_p   → Just (swaplabels i'_q)    (DeferPull2)
  (Just i'_p, _)         → Just i'_p                 (Run1)
  (_, Just i'_q)         → Just (swaplabels i'_q)    (Run2)
  (Nothing, Nothing)     → Nothing                   (Deadlock)
```

Figure 11: Fusion step coordination for a pair of processes.

states. As we discover reachable states we add them to a map *bs* of joint label to the corresponding instruction, and repeat the process to a fixpoint where no new states can be discovered.

Figure 11 defines function *tryStepPair* which decides which process to advance. It starts by calling *tryStep* for both processes. If both can advance, we use heuristics to decide which one to run first.

Clauses (PreferJump1) and (PreferJump2) prioritize processes that can perform a `jump`. This helps collect `jump` instructions together so they are easier for post-fusion optimization to handle (§5.3). The instruction for the second process was computed by calling *tryStep* with the label arguments swapped, so in (PreferJump2) we need to swap the labels back with *swaplabels* (from Figure 13).

Similarly, clauses (DeferPull1) and (DeferPull2) defer `pull` instructions: if one of the instructions is a `pull`, we advance the other one. We do this because `pull` instructions may block, while other instructions are more likely to produce immediate results.

Clauses (Run1) and (Run2) apply when the above heuristics do not apply, or only one of the processes can advance.

Clause (Deadlock) applies when neither process can advance, in which case the processes cannot be fused together and fusion fails.

Figure 12 defines function *tryStep* which schedules a single instruction. This function takes the map of channel types, along with the current label and associated instruction of the first (left) process, and the current label of the other (right) process.

Clause (LocalJump) applies when the left process wants to jump. In this case, the result instruction simply performs the corresponding jump, leaving the right process where it is.

Clause (LocalCase) is similar, except there are two *Next* labels.

Clause (LocalPush) applies when the left process wants to push to a non-shared output channel. In this case the push can be performed directly, with no additional coordination required.

Clause (SharedPush) applies when the left process wants to push to a shared channel. Pushing to a shared channel requires the downstream process to be ready to accept the value at the same time. We encode this constraint by requiring the static input state of the downstream channel to be $\mathtt{none}_F$. When this is satisfied, the result instruction stores the pushed value in the stream buffer variable (`chan` c) and sets the static input state to $\mathtt{pending}_F$, which indicates that the new value is now available.

Still in Figure 12, clause (LocalPull) applies when the left process wants to pull from a local channel, which requires no coordination.

Clause (SharedPull) applies when the left process wants to pull from a shared channel that the other process either pulls from or pushes to. We know that there is already a value in the stream buffer variable, because the state for that channel is $\mathtt{pending}_F$. The result instruction copies the value from the stream buffer variable into a variable specific to the left source process, and the corresponding $\mathtt{have}_F$ channel state in the result label records that it has done so.

Clause (SharedPullInject) applies when the left process wants to pull from a shared channel that both processes pull from, and neither already has a value. The result instruction is a `pull` that loads the stream buffer variable.

Clause (LocalDrop) applies when the left process wants to drop the current value that it read from an unshared input channel, which requires no coordination.

Clause (ConnectedDrop) applies when the left process wants to drop the current value that it received from an upstream process. As the value will have been sent via a heap variable instead of a still extant channel, the result instruction just performs a `jump` while updating the static channel state.

Clauses (SharedDropOne) and (SharedDropBoth) apply when the left process wants to drop from a channel shared by both processes. In (SharedDropOne) the channel states reveal that the other process is still using the value. In this case the result is a `jump` updating the channel state to note that the left process has dropped. In (SharedDropBoth) the channel states reveal that the other process no longer needs the value. In this case the result is a real `drop`, because we are sure that neither process requires the value any longer.

Clause (Blocked) returns `Nothing` when no other clauses apply, meaning that this process is waiting for the other process to advance.

4.1 Fusibility

When we fuse a pair of processes we commit to a particular interleaving of instructions from each process. When we have at least three processes to fuse, the choice of which two to handle first can determine whether this fused result can then be fused with the third process. Consider the following example, where `alt2` pulls two elements from its first input stream, then two from its second, before pushing all four to its output.

```
alternates : S Nat -> S Nat -> S Nat -> S (Nat, Nat)
alternates sInA sInB sInC
 = let  s1   = alt2 sInA sInB
        s2   = alt2 sInB sInC
        sOut = zip s1 s2
   in   sOut
```

If we fuse the two `alt2` processes together first, then try to fuse this result process with the downstream `zip` process, the final fusion transform fails. This happens because the first fusion transform commits to a sequential instruction interleaving where two output values *must* be pushed to stream `s1` first, before pushing values to `s2`. On the other hand, `zip` needs to pull a *single* value from each of its inputs alternately.

Dynamically, if we were to execute the first fused result process, and the downstream `zip` process concurrently, then the execution would deadlock. Statically, when we try to fuse the result process with the downstream `zip` process the deadlock is discovered and fusion fails. Deadlock happens when neither process can advance to the next instruction, and in the fusion algorithm this manifests as the failure of the *tryStepPair* function from Figure 11. The *tryStepPair* function determines which instruction from either process to execute next, and when execution is deadlocked there are none. Fusibility is an under-approximation for *deadlock freedom* of the network.

In practice, the likelihood of fusion succeeding depends on the particular dataflow network. For fusion of pipelines of standard combinators such as `map`, `fold`, `filter`, `scan` and so on, fusion always succeeds. The process implementations of each of these combinators only pull one element at a time from their source streams, before pushing the result to the output stream, so there is no possibility of deadlock. Deadlock can only happen when multiple streams fan-in to a process with multiple inputs, such as with `merge`.

When the dataflow network has a single output stream then we use the method of starting from the process closest to the output stream, walking towards the input streams, and fusing in successive processes as they occur. This allows the interleaving of the intermediate fused process to be dominated by the consumers, rather than producers, as consumers are more likely to have multiple input channels which need to be synchronized. In the worst case the fall back approach is to try all possible orderings of processes to fuse.

$$\begin{array}{lll}
\multicolumn{3}{l}{\mathit{tryStep}\ :\ (\mathit{Channel} \mapsto \mathit{ChannelType2}) \to \mathit{LabelF} \to \mathit{Instruction} \to \mathit{LabelF} \to \mathit{Maybe\ Instruction}}\\
\multicolumn{3}{l}{\mathit{tryStep}\ cs\ (l_p, s_p)\ i_p\ (l_q, s_q)\ =\ \texttt{match}\ i_p\ \texttt{with}}\\
\quad \texttt{jump}\ (l', u') & \to \texttt{Just (jump}\ \ ((l',\ s_p),\ (l_q,\ s_q),\ u')) & \text{(LocalJump)}\\
\quad \texttt{case}\ e\ (l'_t, u'_t)\ (l'_f, u'_f) & \to \texttt{Just (case}\ e\ \ ((l'_t,\ s_p),\ (l_q,\ s_q),\ u'_t)\ ((l'_f,\ s_p),\ (l_q,\ s_q),\ u'_f)) & \text{(LocalCase)}\\
\quad \texttt{push}\ c\ e\ (l', u') & & \\
\qquad \mid cs[c] = \texttt{out1} & \to \texttt{Just (push}\ c\ e\ ((l',\ s_p),\ (l_q,\ s_q),\ u')) & \text{(LocalPush)}\\
\qquad \mid cs[c] = \texttt{in1out1}\ \wedge\ s_q[c] = \texttt{none}_F & \to \texttt{Just (push}\ c\ e\ ((l',\ s_p),\ (l_q,\ s_q[c\ \mapsto\ \texttt{pending}_F]),\ u'[\texttt{chan}\ c\ \mapsto\ e])) & \text{(SharedPush)}\\
\quad \texttt{pull}\ c\ x\ (l', u') & & \\
\qquad \mid cs[c] = \texttt{in1} & \to \texttt{Just (pull}\ c\ x\ ((l',\ s_p),\ (l_q,\ s_q),\ u')) & \text{(LocalPull)}\\
\qquad \mid (cs[c] = \texttt{in2} \vee cs[c] = \texttt{in1out1})\ \wedge\ s_p[c] = \texttt{pending}_F & & \\
\qquad \to \texttt{Just (jump}\ ((l',\ s_p[c\ \mapsto\ \texttt{have}_F]),\ (l_q,\ s_q),\ u'[x\ \mapsto\ \texttt{chan}\ c])) & & \text{(SharedPull)}\\
\qquad \mid cs[c] = \texttt{in2}\ \wedge\ s_p[c] = \texttt{none}_F\ \wedge\ s_q[c] = \texttt{none}_F & & \\
\qquad \to \texttt{Just (pull}\ c\ \texttt{(chan}\ c)\ ((l_p,\ s_p[c\ \mapsto\ \texttt{pending}_F]),\ (l_q,\ s_q[c\ \mapsto\ \texttt{pending}_F]),\ [])) & & \text{(SharedPullInject)}\\
\quad \texttt{drop}\ c\ (l', u') & & \\
\qquad \mid cs[c] = \texttt{in1} & \to \texttt{Just (drop}\ c\ \ ((l',\ s_p),\ (l_q,\ s_q),\ u')) & \text{(LocalDrop)}\\
\qquad \mid cs[c] = \texttt{in1out1} & \to \texttt{Just (jump}\ \ ((l',\ s_p[c\ \mapsto\ \texttt{none}_F]),\ (l_q,\ s_q),\ u')) & \text{(ConnectedDrop)}\\
\qquad \mid cs[c] = \texttt{in2}\ \wedge\ (s_q[c] = \texttt{have}_F \vee s_q[c] = \texttt{pending}_F) & \to \texttt{Just (jump}\ \ ((l',\ s_p[c\ \mapsto\ \texttt{none}_F]),\ (l_q,\ s_q),\ u')) & \text{(SharedDropOne)}\\
\qquad \mid cs[c] = \texttt{in2}\ \wedge\ s_q[c] = \texttt{none}_F & \to \texttt{Just (drop}\ c\ \ ((l',\ s_p[c\ \mapsto\ \texttt{none}_F]),\ (l_q,\ s_q),\ u')) & \text{(SharedDropBoth)}\\
\quad \_\quad \mid \texttt{otherwise} & \to \texttt{Nothing} & \text{(Blocked)}
\end{array}$$

Figure 12: Fusion step for a single process of the pair.

$$\begin{array}{lcll}
\mathit{channels} & : & \multicolumn{2}{l}{\mathit{Process} \to \mathit{Process} \to (\mathit{Channel} \mapsto \mathit{ChannelType2})}\\
\mathit{channels}\ p\ q & = & \{c = \texttt{in2} & \mid c \in (\texttt{ins}\ p\ \cap\ \texttt{ins}\ q)\}\\
 & \cup & \{c = \texttt{in1} & \mid c \in (\texttt{ins}\ p\ \cup\ \texttt{ins}\ q)\ \wedge\ c\ \notin (\texttt{outs}\ p\ \cup\ \texttt{outs}\ q)\}\\
 & \cup & \{c = \texttt{in1out1} & \mid c \in (\texttt{ins}\ p\ \cup\ \texttt{ins}\ q)\ \wedge\ c\ \in (\texttt{outs}\ p\ \cup\ \texttt{outs}\ q)\}\\
 & \cup & \{c = \texttt{out1} & \mid c \notin (\texttt{ins}\ p\ \cup\ \texttt{ins}\ q)\ \wedge\ c\ \in (\texttt{outs}\ p\ \cup\ \texttt{outs}\ q)\}
\end{array}$$

$$\begin{array}{lcl}
\mathit{outlabels}\ :\ \mathit{Instruction} \to \{\mathit{Label}\} & & \\
\mathit{outlabels}\ (\texttt{pull}\ c\ x\ (l, u)) & = & \{l\}\\
\mathit{outlabels}\ (\texttt{drop}\ c\ (l, u)) & = & \{l\}\\
\mathit{outlabels}\ (\texttt{push}\ c\ e\ (l, u)) & = & \{l\}\\
\mathit{outlabels}\ (\texttt{case}\ e\ (l, u)\ (l', u')) & = & \{l, l'\}\\
\mathit{outlabels}\ (\texttt{jump}\ (l, u)) & = & \{l\}
\end{array}
\qquad
\begin{array}{lcl}
\mathit{swaplabels}\ :\ \mathit{Instruction} \to \mathit{Instruction} & & \\
\mathit{swaplabels}\ (\texttt{pull}\ c\ x\ ((l_1, l_2), u)) & = & \texttt{pull}\ c\ x\ ((l_2, l_1), u)\\
\mathit{swaplabels}\ (\texttt{drop}\ c\ ((l_1, l_2), u)) & = & \texttt{drop}\ c\ ((l_2, l_1), u)\\
\mathit{swaplabels}\ (\texttt{push}\ c\ e\ ((l_1, l_2), u)) & = & \texttt{push}\ c\ e\ ((l_2, l_1), u)\\
\mathit{swaplabels}\ (\texttt{case}\ e\ ((l_1, l_2), u)\ ((l'_1, l'_2), u')) & = & \texttt{case}\ e\ ((l_2, l_1), u)\ ((l'_2, l'_1), u')\\
\mathit{swaplabels}\ (\texttt{jump}\ ((l_1, l_2), u)) & = & \texttt{jump}\ ((l_2, l_1), u)
\end{array}$$

Figure 13: Utility functions

5 IMPLEMENTATION

Stream fusion is ultimately performed for practical reasons: we want the fused result program to run faster than the original unfused program.

5.1 Finite streams

The processes we have seen so far deal with infinite streams, but in practice most streams are finite. Certain combinators such as `fold` and append only make sense on finite streams, and others like `take` produce inherently finite output. We have focussed on the infinite stream version as it is simpler to explain and prove, but supporting finite streams does not require substantial conceptual changes.

Unlike infinite streams, pulling from a finite stream can fail, meaning the stream is finished. We therefore modify the `pull` instruction to have two output labels: one to execute when a value is pulled, and the other to execute when the stream is finished. On the pushing end, we also need some way of finishing streams, so we add a new instruction to close an output stream.

During evaluation we need some way of knowing whether a stream is closed, which can be added as an extra constructor in the *InputState* type. The same constructor is added to the static input state used by fusion. In this way, for any changes made to evaluation, the analogous static change must be made in the fusion transform.

It is also possible to encode finite streams as infinite streams with an explicit end-of-stream marker (EOF) and case statements. However, this requires the fusion transform to reason about case statements' predicates. By making the structure of finite streams explicit and constraining how processes use finite streams, it is not necessary to rely on heuristics for deciding equality of predicates.

This finite stream extension is described in more detail in the appendix of the extended version of this paper, which is available at http://cse.unsw.edu.au/~amosr/papers/merges.pdf.

5.2 Benchmarks

We have implemented this system using Template Haskell in a library called `folderol`[1]. To show practical examples, we use the finite stream extension mentioned in §5.1. We present three benchmarks: two file-based benchmarks, and one array algorithm.

For the file benchmarks, we compare against three Haskell streaming libraries: 'Conduit', 'Pipes', and 'Streaming'. These streaming libraries are pull-based, and do not naturally support multiple outputs: the split in the dataflow graph must be hand-fused, or somehow rewritten as a straight-line computation. These libraries also have a monadic interface, which allows the structure of the dataflow graph to depend on the values. This expressiveness has a price: if the dataflow graph can change dynamically, we cannot statically fuse it.

The first file benchmark simply appends two files while counting the lines. In Pipes and Conduit, counting the lines is implemented as a pipe which counts each line before passing it along. The first group in Figure 14 shows the runtimes for appending 2MB of data.

The second file benchmark takes a file and partitions it into two files: one with even-length lines, and one with odd-length lines. The output lines are also counted. Even with partial hand-fusion because of the multiple outputs, the Pipes and Conduit programs are slower than ours, as well as losing the abstraction benefits from using a high-level library. The 'Streaming' library allows streams to be shared in a fairly straightforward way and does not require hand-fusion, but is also the slowest in this benchmark. The second group in Figure 14 shows the runtimes for partitioning a 1MB file.

Quickhull is a divide-and-conquer spatial algorithm to find the smallest convex hull containing all points. At its core is an operation called 'filterMax' which takes a line and an array of points, and finds the farthest point above the line, as well as all points above the line.

Here we also compare against a `Data.Vector` program, which uses shortcut fusion. The shortcut fusion system cannot fuse both operations into a single loop, and both operations must recompute the distances between the line and each point. As before, the Conduit and Pipes programs must be partially hand-fused. The third group in Figure 14 shows the runtimes for Quickhull over 40MB of data, or half a million points, while the final group uses more data (120MB) to compare directly against `Data.Vector`.

5.3 Optimisation and Drop Instructions

After we have fused two processes together, it may be possible to simplify the result before fusing in a third. Consider the result of fusing `group` and `merge` which we saw back in Figure 4. At labels `F1` and `F2` are two consecutive `jump` instructions. The update expressions attached to these instructions are also non-interfering, which means we can safely combine these instructions into a single `jump`. In general, we prefer to have `jump` instructions from separate processes scheduled into consecutive groups, rather than spread out through the result code. The (PreferJump) clauses of Figure 11 implement a heuristic that causes jump instructions to be scheduled before all others, so they tend to end up in these groups.

Other `jump` instructions like the one at `F5` have no associated update expressions, and thus can be eliminated completely. Another simple optimization is to perform constant propagation, which in this case would allow us to eliminate the first `case` instruction.

[1]https://github.com/amosr/folderol

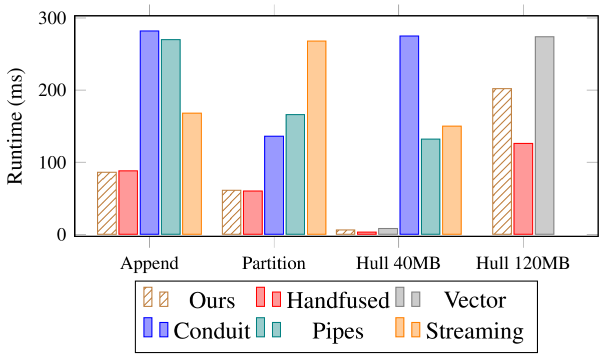


Figure 14: Runtime for benchmarks; lower is faster.

Minimising the number of states in an intermediate process has the follow-on effect that the final fused result also has fewer states. Provided we do not change the order of instructions that require synchronization with other processes (`pull`, `push` or `drop`), the fusibility of the overall process network will not be affected.

When the two processes are able to accept the next variable from the stream at the same time, there is no need for the separate stream buffer variable. This is the case in Figure 4, and we can perform a copy-propagation optimisation, replacing all occurrences of `v` and `x1` with the single variable `b1`. To increase the chance that we can perform copy-propagation, we need both processess to want to pull from the same stream at the same time. Moving the `drop` instruction for a particular stream as late as possible prevents a `pull` instruction from a second process being scheduled in too early.

6 PROOFS

Our fusion system is formalized in Coq, and we have proved soundness of *fusePair*: if the fused result process produces a particular sequence of values on its output channels then the two source processes may also produce that same sequence. The converse is not true, however: concurrent evaluation represents all possible interleavings, of which the fused process is only one.

```
Theorem Soundness
  (P1 : Program L1 C V1) (P2 : Program L2 C V2)
  (ss : Streams) (h : Heap) (l1 : L1) (l2 : L2)
  (is1 : InputStates) (is2 : InputStates)
  :  EvalBs (fuse P1 P2) ss h (LX l1 l2 is1 is2)
  -> EvalOriginal Var1 P1 P2 is1 ss h l1
  /\ EvalOriginal Var2 P2 P1 is2 ss h l2.
```

`EvalBs` evaluates the fused program, and `EvalOriginal` ensures that the original program evaluates with that program's subset of the result heap. The Coq formalization has some small differences from the system in this paper. Instead of implementing non-deterministic evaluation we sequentially evaluate each source processes independently, and compare the output values to the ones produced by sequential evaluation of the fused result process. This is sufficient for our purposes because we are mainly interested in the value correctness of the fused program, rather than making a statement about the possible execution orders of the source processes when run concurrently.

7 RELATED WORK

This work aims to address the limitations of current combinator-based array fusion systems. As stated in the introduction, neither pull-based or push-based fusion is sufficient. Some combinators are inherently push-based, particularly those with multiple outputs such as `unzip`; while others are inherently pull-based, such as `zip`.

However, short cut fusion [11] relies on inlining which, like pull-based streams, only occurs when there is a single consumer. Push-based short cut fusion systems *do* exist, notably the original foldr/build formulation, but support neither `zip` nor `unzip` [20, 27].

Recent work on stream fusion [17] uses staged computation in a push-based system to ensure all combinators are inlined. When streams are used multiple times this causes excessive inlining, which duplicates work. This can change the semantics for effectful streams.

Data flow fusion [20] is neither pull-based nor push-based, and supports arbitrary splits and joins. It supports only a fixed set of standard combinators such as `map`, `filter` and `fold`, and converts each stream to a series with explicit rate types, similar to the clock types of Lucid Synchrone [2].

One way to address the difference between pull and push streams is to explicitly support both [3, 22]. Here, pull streams have the type `Source` and represent a source, always ready to be pulled, while push streams have the type `Sink` and represent a sink, always ready to accept data. The addition of push streams allows more programs to be fused than pull-only systems, but the computation must be manually split into sources and sinks.

The duality between pull and push arrays has also been explored in Obsidian [8, 28]. Here the distinction is made for the purpose of code generation for GPUs rather than fusion, as operations such as appending pull arrays require conditionals inside the loop, whereas using push arrays moves these conditionals outside the loop.

Meta-repa [1] supports both array types in a similar way, using Template Haskell for code generation. It supports fusion on both array types. When arrays are used multiple times, they must be explicitly reified with a 'force' operation to avoid duplication.

Streaming IO libraries have blossomed in the Haskell ecosystem, generally based on Iteratees [16]. Libraries such as `conduit` [25], `enumerator` [24], `machines` [18] and `pipes` [12] are all designed to write stream computations with bounded buffers, but do not guarantee fusion.

In relation to process calculi, synchronised product has been suggested as a method for fusing Kahn process networks together [10], but does not appear to have been implemented or evaluated. The synchronised product of two processes allows either process to take independent or local steps at any time, but shared actions, such as when both processes communicate on the same channel, must be taken in both processes at the same time. When two processes share multiple channels, synchronised product will fail unless both processes read the channels in exactly the same order. In our system the use of stream buffer variables allows some leeway in when processes must take shared steps.

Synchronous languages such as LUSTRE [13], Lucy-n [23] and SIGNAL [19] all use some form of clock calculus and causality analysis to ensure that programs can be statically scheduled with bounded buffers [7]. These languages describe *passive* processes where values are fed in to streams from outside environments, such as data coming from sensors. In contrast, our processes are *active* and have control over when data is pulled from the source streams, which is nessesary for multiple input combinators such as `merge` and `append`. Synchronous dataflow languages reject operators with value dependent control flow such as `merge`, while general dataflow languages fall back on less performant dynamic scheduling [4].

Synchronous dataflow (SDF; not to be confused with synchronous languages above) is a dataflow graph model of computation where each node has constant, statically known input and output rates. StreamIt [29] uses synchronous dataflow for scheduling when possible, otherwise falling back to dynamic scheduling [26]. Boolean dataflow and integer dataflow [5, 6] extend SDF with boolean and integer valued control ports, but only support limited control flow structures.

REFERENCES

[1] Johan Ankner and Josef David Svenningsson. 2013. An EDSL approach to high performance Haskell programming. In *ACM SIGPLAN Notices*.

[2] Albert Benveniste, Paul Caspi, Stephen A Edwards, Nicolas Halbwachs, Paul Le Guernic, and Robert De Simone. 2003. The synchronous languages 12 years later. *Proc. IEEE* (2003).

[3] Jean-Philippe Bernardy and Josef Svenningsson. 2015. On the Duality of Streams How Can Linear Types Help to Solve the Lazy IO Problem?. In *IFL*.

[4] Adnan Bouakaz. 2013. *Real-time scheduling of dataflow graphs*. Ph.D. Dissertation. Université Rennes 1.

[5] Joseph T Buck. 1994. Static scheduling and code generation from dynamic dataflow graphs with integer-valued control streams. In *ACSSC*.

[6] Joseph Tobin Buck and Edward A Lee. 1993. Scheduling dynamic dataflow graphs with bounded memory using the token flow model. In *ICASSP*.

[7] Paul Caspi and Marc Pouzet. 1996. Synchronous Kahn Networks. In *ICFP: International Conference on Functional Programming*.

[8] Koen Claessen, Mary Sheeran, and Joel Svensson. 2012. Expressive array constructs in an embedded GPU kernel programming language. In *DAMP*.

[9] Duncan Coutts, Roman Leshchinskiy, and Don Stewart. 2007. Stream fusion: From lists to streams to nothing at all. In *SIGPLAN*.

[10] Pascal Fradet and Stéphane Hong Tuan Ha. 2004. Network fusion. In *APLAS*.

[11] Andrew Gill, John Launchbury, and Simon L Peyton Jones. 1993. A short cut to deforestation. In *FPCA*.

[12] Gabriel Gonzalez. 2012. http://hackage.haskell.org/package/pipes. (2012).

[13] Nicholas Halbwachs, Paul Caspi, Pascal Raymond, and Daniel Pilaud. 1991. The synchronous data flow programming language LUSTRE. *IEEE* (1991).

[14] Gilles Kahn, David MacQueen, and others. 1976. Coroutines and networks of parallel processes. (1976).

[15] Michael Kay. 2009. You pull, I'll push: on the polarity of pipelines. In *Balisage*.

[16] Oleg Kiselyov. 2012. Iteratees. In *FLOPS*.

[17] Oleg Kiselyov, Aggelos Biboudis, Nick Palladinos, and Yannis Smaragdakis. 2017. Stream Fusion, to Completeness. In *POPL*.

[18] Edward Kmett, Rúnar Bjarnason, and Josh Cough. 2012. http://hackage.haskell.org/package/machines. (2012).

[19] Paul Le Guernic, Jean-Pierre Talpin, and Jean-Christophe Le Lann. 2003. Polychrony for system design. *Journal of Circuits, Systems, and Computers* (2003).

[20] Ben Lippmeier, Manuel MT Chakravarty, Gabriele Keller, and Amos Robinson. 2013. Data flow fusion with series expressions in Haskell. In *SIGPLAN*.

[21] Ben Lippmeier, Manuel M. T. Chakravarty, Gabriele Keller, and Simon L. Peyton Jones. 2012. Guiding parallel array fusion with indexed types. In *Haskell*.

[22] Ben Lippmeier, Fil Mackay, and Amos Robinson. 2016. Polarized data parallel data flow. In *FHPC*.

[23] Louis Mandel, Florence Plateau, and Marc Pouzet. 2010. Lucy-n: a n-synchronous extension of Lustre. In *MPC*.

[24] John Millikin and Mikhail Vorozhtsov. 2011. http://hackage.haskell.org/package/enumerator. (2011).

[25] Michael Snoyman. 2011. http://hackage.haskell.org/package/conduit. (2011).

[26] Robert Soule, Michael I. Gordon, Saman Amarasinghe, Robert Grimm, and Martin Hirzel. 2013. Dynamic Expressivity with Static Optimization for Streaming Languages. In *DEBS*.

[27] Josef Svenningsson. 2002. Shortcut fusion for accumulating parameters & zip-like functions. In *SIGPLAN*.

[28] Bo Joel Svensson and Josef Svenningsson. 2014. Defunctionalizing push arrays. In *FHPC*.

[29] William Thies, Michal Karczmarek, and Saman Amarasinghe. 2002. StreamIt: A language for streaming applications. In *Compiler Construction*.

A FINITE STREAMS

This appendix briefly describes the finite streams extension, showing the changes to instructions and the fusion algorithm. The finite evaluation rules are not shown, as the changes follow the same structure as the changes to the fusion algorithm.

Figure 15 shows the grammar for instructions and the static input state. The first group of instructions containing `push`, `drop`, `case` and `jump` are unchanged.

The `pull` instruction is modified to have two output labels, similar to `case`. The first, the success branch, is used when the input stream is still open and pulling succeeds, in which case the variable is set to the pulled value as before. The second output label, the closed branch, is used when the input stream has been closed, and the variable is not written to. This new `pull` is analogous to a `pull` followed by a `case` in the infinite stream version.

The `close` instruction is used by a pushing process to close or end an output stream. Any subsequent pulls from this channel in other processes will take the closed branch. After an output channel is closed, it cannot be pushed to and remains closed forever.

Finally, the `exit` instruction is used once a process is finished with all its streams, and has nothing left to do. All output streams must be closed before the process finishes. This instruction has no output labels, as there is nothing further to execute.

Also in Figure 15, the static input state used for fusion (*InputStateF*) must now track closed streams. The new constructor $\texttt{closed}_F$ denotes that the stream is closed, while the rest is unchanged.

For the fusion algorithm, the top-level function *fusePair* remains unchanged. The functions *outlabels* and *swaplabels* are not shown as they are easily modified by adding cases for the new instructions.

Figure 16 shows the modified *tryStepPair* function. This function uses the same heuristics to decide which process to execute when both can progress, but now that the processes can finish with `exit`, we must take care to only finish the fused process once *both* source processes are finished. The (DeferExit1) and (DeferExit2) clauses achieve this by forcing the other process to run if one is an `exit`. Once both processes are finished, both new clauses will fail while (Run1) succeeds, using the `exit` from the first process. Another way to think of this is that if either process has work to do, the fused process still has work to do.

Figure 17 shows the modified *tryStep* function. The clauses for the unchanged instructions `push`, `drop`, `case` and `jump` remain unchanged; these are reordered to the top of the function.

The `pull` clauses use l'_o for the open output label, and l'_c for the closed label. Clause (LocalPull) now uses two output labels, and leaves the other process as-is.

Clause (SharedPull) applies when the channel state is `pending`, meaning there is already a value available. This means that the channel is not yet closed, and the success branch can be taken.

Clause (SharedPullInject) applies when both processes need to pull from a shared input. As before, we execute a real `pull`, this time with two branches. In the success branch, the input states are set to `pending` as before. In the closed branch, the input states are set to `closed` so the next and subsequent pulls take the closed branch.

Clause (SharedPullClosed) applies when the channel state is `closed`, which means either the other process has pulled and discovered that the channel is closed, or in case of connected input, the other process has closed the channel. Either way we simply jump, taking the closed branch of the `pull`.

Clause (LocalClose) applies when closing a local output.

Clause (SharedClose) applies when closing a connected output. As with (SharedPush), the other input state for the other process must be empty and ready to pull from the channel. The input state for the other process is then set to `closed`, forcing its next pull to take the closed branch.

Finally, clause (LocalExit) allows the process to finish. However, recall that the *tryStepPair* function has been modified to only `exit` when both processes are ready to finish.

```
Instruction  ::= push   Channel Exp       Next
               | drop   Channel           Next
               | case   Exp      Next     Next
               | jump                     Next

               | pull   Channel Variable  Next Next
               | close  Channel           Next
               | exit

InputStateF  ::= none_F | pending_F | have_F | closed_F
```

Figure 15: Finite instructions

```
tryStepPair : (Channel ↦ ChannelType2)
            → LabelF → Instruction → LabelF → Instruction
            → Maybe Instruction
tryStepPair cs l_p i_p l_q i_q =
  match (tryStep cs l_p i_p l_q, tryStep cs l_q i_q l_p) with
  (Just i'_p, Just i'_q)
   | exit ← i'_q          → Just i'_p                 (DeferExit1)
   | exit ← i'_p          → Just (swaplabels i'_q)    (DeferExit2)
   | jump _ ← i'_p        → Just i'_p                 (PreferJump1)
   | jump _ ← i'_q        → Just (swaplabels i'_q)    (PreferJump2)
   | pull _ _ _ ← i'_q    → Just i'_p                 (DeferPull1)
   | pull _ _ _ ← i'_p    → Just (swaplabels i'_q)    (DeferPull2)
  (Just i'_p, _)          → Just i'_p                 (Run1)
  (_, Just i'_q)          → Just (swaplabels i'_q)    (Run2)
  (Nothing, Nothing)      → Nothing                   (Deadlock)
```

Figure 16: Fusion step coordination for a pair of processes.

```
$tryStep\ :\ (Channel\ \mapsto\ ChannelType2) \to LabelF \to Instruction \to LabelF \to Maybe\ Instruction$
$tryStep\ cs\ (l_p, s_p)\ i_p\ (l_q, s_q)$ = match $i_p$ with
  jump $(l', u')$                        → Just (jump $((l',\ s_p),\ (l_q,\ s_q),\ u'))$                                   (LocalJump)
  case $e\ (l'_t, u'_t)\ (l'_f, u'_f)$    → Just (case $e\ ((l'_t,\ s_p),\ (l_q,\ s_q),\ u'_t)\ ((l'_f,\ s_p),\ (l_q,\ s_q),\ u'_f))$   (LocalCase)
  push $c\ e\ (l', u')$
    | $cs[c]$ = out1
    → Just (push $c\ e\ ((l',\ s_p),\ (l_q,\ s_q),\ u'))$                                                              (LocalPush)
    | $cs[c]$ = in1out1 $\wedge\ s_q[c]$ = none$_F$
    → Just (push $c\ e\ ((l',\ s_p),\ (l_q,\ s_q[c\ \mapsto$ pending$_F]),\ u'[$chan $c\ \mapsto\ e]))$                        (SharedPush)
  drop $c\ (l', u')$
    | $cs[c]$ = in1                                                  → Just (drop $c\ ((l',\ s_p),\ (l_q,\ s_q),\ u'))$                  (LocalDrop)
    | $cs[c]$ = in1out1                                              → Just (jump $((l',\ s_p[c\ \mapsto$ none$_F]),\ (l_q,\ s_q),\ u'))$   (ConnectedDrop)
    | $cs[c]$ = in2 $\wedge\ (s_q[c]$ = have$_F \vee s_q[c]$ = pending$_F)$   → Just (jump $((l',\ s_p[c\ \mapsto$ none$_F]),\ (l_q,\ s_q),\ u'))$   (SharedDropOne)
    | $cs[c]$ = in2 $\wedge\ s_q[c]$ = none$_F$                        → Just (drop $c\ ((l',\ s_p[c\ \mapsto$ none$_F]),\ (l_q,\ s_q),\ u'))$   (SharedDropBoth)

  pull $c\ x\ (l'_o, u'_o)\ (l'_c, u'_c)$
    | $cs[c]$ = in1
    → Just (pull $c\ x\ ((l'_o,\ s_p),\ (l_q,\ s_q),\ u'_o)\ ((l'_c,\ s_p),\ (l_q,\ s_q),\ u'_c))$                                 (LocalPull)
    | $(cs[c]$ = in2 $\vee\ cs[c]$ = in1out1) $\wedge\ s_p[c]$ = pending$_F$
    → Just (jump $((l'_o,\ s_p[c\ \mapsto$ have$_F]),\ (l_q,\ s_q),\ u'_o[x\ \mapsto$ chan $c]))$                                   (SharedPull)
    | $cs[c]$ = in2 $\wedge\ s_p[c]$ = none$_F\ \wedge\ s_q[c]$ = none$_F$
    → Just (pull $c$ (chan $c$) $((l_p,\ s_p[c\ \mapsto$ pending$_F]),\ (l_q,\ s_q[c\ \mapsto$ pending$_F]),\ [])$
                                $((l_p,\ s_p[c\ \mapsto$ closed$_F]),\ (l_q,\ s_q[c\ \mapsto$ closed$_F]),\ []))$                (SharedPullInject)
    | $(cs[c]$ = in2 $\vee\ cs[c]$ = in1out1) $\wedge\ s_p[c]$ = closed$_F$
    → Just (jump $((l'_c,\ s_p),\ (l_q,\ s_q),\ u'_c))$                                                                  (SharedPullClosed)
  close $c\ (l', u')$
    | $cs[c]$ = out1                                 → Just (close $c\ ((l',\ s_p),\ (l_q,\ s_q),\ u'))$                           (LocalClose)
    | $cs[c]$ = in1out1 $\wedge\ s_q[c]$ = none$_F$     → Just (close $c\ ((l',\ s_p),\ (l_q,\ s_q[c\ \mapsto$ closed$_F]),\ u'))$     (SharedClose)
  exit                                   → Just exit                                                                     (LocalExit)
  _   | otherwise                        → Nothing                                                                       (Blocked)
```

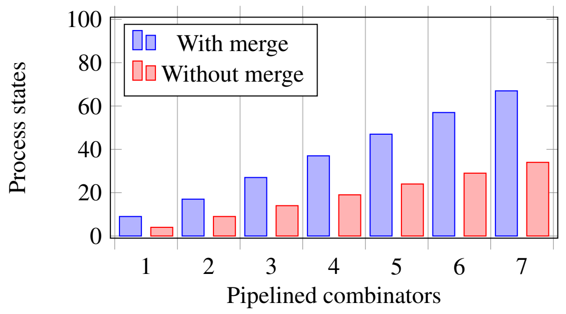
Figure 17: Fusion step for a single process of the pair.

B RESULT SIZE

As with any fusion system, we must be careful that the size of the result code does not become too large when more and more processes are fused together.

B.1 Fusing Pipelines of Processes

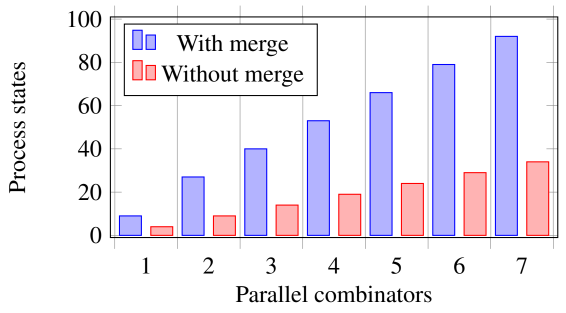
The following figure shows the maximum number of output states in the result when a particular number of processes are fused together in a pipelined-manner.



To produce the above graph we programmatically generated dataflow networks for *all possible* pipelined combinations of the `map`, `filter`, `scan`, `group` and `merge` combinators, and tried all possible fusion orders consiting of adjacent pairs of processes. The `merge` combinator itself has two inputs, so only works at the very start of the pipeline — we present result for pipelines with and without a `merge` at the start.

B.2 Fusing Parallel Processes

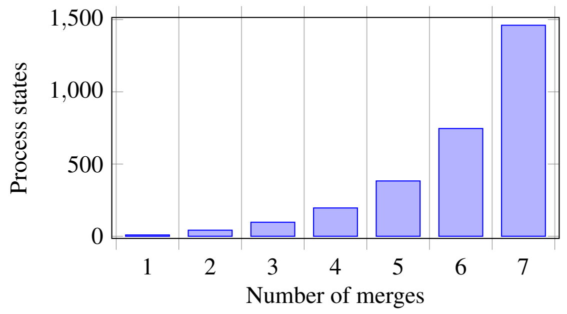
The following figure shows the number of states in the result when the various combinations of combinators are fused in parallel, for example, we might have a `map` and a `filter` processing the same input stream. In both cases the number of states in the result process grows linearly with the number of processes. In all combinations, with up to 7 processes there are less than 100 states in the result process.



The size of the result process is roughly what one would get when inlining the definitions of each of the original source processes. This is common with other systems based on inlining and/or template meta-programming, and is not prohibitive.

B.3 Fusing Merges

On the other hand, the following figure shows the results for a pathological case where the size of the output program is exponential in the number of input processes. The source dataflow networks consists of N merge processes, N+1 input streams, and a single output stream. The output of each merge process is the input of the next, forming a chain of merges. In source notation the network for N = 3 is `sOut = merge sIn1 (merge sIn2 (merge sIn3 sIn4))`.



When fusing two processes the fusion algorithm essentially compares every state in the first process with every state in the second, computing a cross product. During the fusion transform, as states in the result process are generated they are added to a finite map — the `instrs` field of the process definition. The use of the finite map ensures that identical states are always combined, but genuinely different states always make it into the result. In the worst case, fusion of two processes produces $O(n * m)$ different states, where n and m are the number of states in each. If we assume the two processes have about the same number of states then this is $O(n^2)$. Fusing the next process into this result yields $O(n^3)$, so overall the worst case number of states in the result will be $O(n^k)$, where k is the number of processes fused.

In the particular case of `merge`, the implementation has two occurrences of the `push` instruction. During fusion, the states for the consuming process are inlined at each occurrence of `push`. These states are legitimately different because at each occurence of push the input channels of the merge process are in different channel states, and these channel states are included in the overall process state.

C COMBINATORS

Here we show the definitions of some combinators. We start with simple combinators supported by most streaming systems, and progress to more interesting combinators. Some standard combinators such as `fold`, `take` and `append` are missing due to the infinite nature of our streams, but could be implemented with the finite stream extension. The fact that segmented versions of these combinators can be implemented is compelling evidence of this.

Many of these combinators take a "default" argument, which is used to initialise the heap, but the stored value is never actually read. Ideally these could be left unspecified, or the heap left uninitialised in cases where it is never read.

C.1 Map

The map combinator applies a function to every element of the stream. This is some more text.

```
map
 = λ (f : α → β) (default : α)
     (sIn: Stream α) (sOut: Stream β).
   ν (a: α) (L0..L2: Label).
   process
   { ins:    { sIn }
   , outs:   { sOut }
   , heap:   { a = default }
   , label:  L0
   , instrs: { L0 = pull sIn     a  L1 []
             , L1 = push sOut (f a) L2 []
             , L2 = drop sIn        L0 [] } }
```

C.2 Filter

Filter returns a new stream containing only the elements that satisfy some predicate.

```
filter
 = λ (f : α → Bool) (default : α)
     (sIn: Stream α) (sOut: Stream α).
   ν (a: α) (L0..L3: Label).
   process
   { ins:    { sIn }
   , outs:   { sOut }
   , heap:   { a = default }
   , label:  L0
   , instrs: { L0 = pull sIn  a     L1 []
             , L1 = case   (f a)    L2 []  L3 []
             , L2 = push sOut a     L3 []
             , L3 = drop sIn        L0 [] } }
```

C.3 Partition

Partition is similar to filter, but has two output streams: those that satisfy the predicate, and those that do not. Partition is an inherently push-based operation, and cannot be supported by pull streams without buffering.

```
partition
 = λ (f : α → Bool) (default : α)
     (sIn:   Stream α)
     (sOut1: Stream α) (sOut2: Stream α).
   ν (a: α) (L0..L4: Label).
   process
   { ins:    { sIn }
   , outs:   { sOut1, sOut2 }
   , heap:   { a = default }
   , label:  L0
   , instrs: { L0 = pull sIn   a    L1 []
             , L1 = case    (f a)   L2 []  L3 []
             , L2 = push sOut1 a    L4 []
             , L3 = push sOut2 a    L4 []
             , L4 = drop sIn        L0 [] } }
```

C.4 Zip

Zip, or zip-with, pairwise combines two input streams. Zipping is an inherently pull-based operation.

```
zipWith
 = λ (f : α → β → γ) (default1 : α) (default2 : β)
     (sIn1: Stream α) (sIn2: Stream β)
     (sOut: Stream γ).
   ν (a: α) (b : β) (L0..L4: Label).
   process
   { ins:    { sIn1, sIn2 }
   , outs:   { sOut }
   , heap:   { a = default1, b = default2 }
   , label:  L0
   , instrs: { L0 = pull sIn1 a       L1 []
             , L1 = pull sIn2 b       L2 []
             , L2 = push sOut (f a b) L3 []
             , L3 = drop sIn1         L4 []
             , L4 = drop sIn2         L0 [] } }
```

C.5 Scan

Scan is similar to a fold, but instead of returning a single value at the end, it returns an intermediate value for each element of the stream.

```
scan
 = λ (k : α → β → β) (z : β) (default : α)
     (sIn: Stream α) (sOut: Stream β).
   ν (a: α) (s : β) (L0..L2: Label).
   process
   { ins:    { sIn  }
   , outs:   { sOut }
   , heap:   { a = default, s = z }
   , label:  L0
   , instrs: { L0 = pull sIn  a   L1 []
             , L1 = push sOut s   L2 [ s = f a s ]
             , L2 = drop sIn      L0 [] } }
```

C.6 Segmented Fold

Segmented fold performs a fold over each nested stream, using a segmented representation. Here we are representing nested streams using one stream for the lengths of each substream, and another stream containing the values. The output stream has the same rate as the lengths stream. It reads a count (c) from the lengths stream, setting the fold state to zero (z). Then it reads count times from the values stream, updating the fold state. Afterwards, it pushes the final fold state, and continues to read a new count.

```
folds
 = λ (k : α → β → β) (z : β) (default : α)
     (sLens: Stream Nat) (sVals: Stream α)
     (sOut:  Stream β).
   ν (c : Nat) (a: α) (s : β) (L0..L5: Label).

   process
   { ins:    { sLens, sVals }
   , outs:   { sOut }
   , heap:   { c = 0, a = default, s = z }
   , label:  L0
   , instrs: { L0 = pull sLens c   L1 [ s = z ]
             , L1 = case (c > 0)   L2 []  L4 []
             , L2 = pull sVals a   L3 []
             , L3 = drop sVals     L1 [ c = c - 1
                                      , s = k s a ]
             , L4 = push sOut  s   L5 []
             , L5 = drop sLens     L0 [] } }
```

C.7 Segmented Take

Segmented take computes an n-length prefix of each nested stream. It starts by reading a count from the lengths stream, then copies at most n elements. If there are leftovers, it pulls and discards them, then pulls the next length.

```
takes
 = λ (n : Nat) (default : α)
     (sLens: Stream Nat) (sVals: Stream α)
     (oLens: Stream Nat) (oVals: Stream α).
   ν (c : Nat) (take : Nat) (ix : Nat) (a: α)
     (L0..L9: Label).

   process
   { ins:    { sLens, sVals }
   , outs:   { oLens, oVals }
   , heap:   { c = 0, take = 0, ix = 0, a = default }
   , label:  L0
   , instrs: { L0 = pull sLens c     L1
                         [ ix = 0, take = min count n ]
             , L1 = push oLens take  L2 []
             , L2 = case (ix < take) L3 []  L6 []
             , L3 = pull sVals a     L4 []
             , L4 = push oVals a     L5 []
             , L5 = drop sVals       L2 [ix = ix+1]
             , L6 = case (ix < c)    L7 []  L9 []
             , L7 = pull sVals a     L8 []
             , L8 = drop sVals       L6 [ix = ix+1]
             , L9 = drop sLens       L0 [] } }
```

C.8 Segmented Append

Segmented append takes two segmented streams as input, and appends each nested stream. It starts by reading a length from both lengths streams into a and b, and pushes the sum of both lengths. It then copies over a elements from the first values stream, then copies over b elements from the second values stream.

```
appends
 = λ (default : α)
     (aLens: Stream Nat) (aVals: Stream α)
     (bLens: Stream Nat) (bVals: Stream α)
     (oLens: Stream Nat) (oVals: Stream α).
   ν (a : Nat) (b : Nat) (v: α) (L0..L12: Label).
   process
   { ins:    { aLens, aVals, bLens, bVals }
   , outs:   { oLens, oVals }
   , heap:   { a = 0, b = 0, v = default }
   , label:  L0
   , instrs: { L0  = pull aLens  a     L1 []
             , L1  = pull bLens  b     L2 []
             , L2  = push oLens (a+b)  L3 []

             , L3  = case (a > 0)      L4 []  L7 []
             , L4  = pull aVals v      L5 []
             , L5  = push oVals v      L6 []
             , L6  = drop aVals        L3 [ a = a-1 ]

             , L7  = case (b > 0)      L8 []  L11[]
             , L8  = pull bVals v      L9 []
             , L9  = push oVals v      L10[]
             , L10 = drop bVals        L7 [ b = b-1 ]

             , L11 = drop aLens         L12[]
             , L12 = drop bLens         L0 [] } }
```